Poisson , Dobinski , Rota and coherent states- a fortieth anniversary memoir

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Forty years ago Rota G. C. [1] while proving that the exponential generating function for Bell numbers B_n is of the form

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}(B_n) = \exp(e^x - 1) \tag{1}$$

used the linear functional L such that

$$L(X^{\underline{n}}) = 1, \qquad n \geq 0 \tag{2}$$

Then Bell numbers (see: formula (4) in [1]) are defined by

$$L(X^n) = B_n, \qquad n \geq 0.$$

Let us notice then that the above formula is exactly the Dobinski formula [2] if L is interpreted as the average functional for the random variable X with the Poisson distribution with $L(X) = 1$. (It is Blissard calculus inspired umbral formula [1]).

Quite recently an interest to Stirling numbers and consequently to Bell numbers was revived among "coherent states physicists" [3, 4]. Namely the expectation value with respect to coherent state $|\gamma >$ with $|\gamma| = 1$ of the n-th power of the number of quanta operator is "just" the n-th Bell number B_n and the explicit formula for this expectation number of quanta is "just" Dobinski formula [3].

One faces the same situation with the q-coherent states case [3] i.e. the expectation value with respect to q-coherent state $|\gamma>$ with $|\gamma| = 1$ of the n-th power of the number operator is the n-th q-Bell number [5] and the explicit formula becomes q-Dobinski formula. Let us notice then that similar new q-Dobinski formula valid for a new q-analogue of Stirling numbers of Cigler [6] might also be interpreted as the average of random variable X_q^n with the same Poisson distribution with $L(X) = 1$ i.e.

$$L(X_q^n) = B_n(q), \qquad n \geq 0; \qquad X_q^n \equiv X(X-1+q)...(X-1+q^{n-1}). \quad (3)$$

For that to see use the identity by Cigler [6]

$$(x+1)(x+q)...(x+q^{n-1}) = \sum_{k=0}^{n} \left\{ {n \atop k} \right\}_q (x+1)^{\underline{k}} \quad (4)$$

These Cigler q-analogue of Stirling numbers $\left\{ {n \atop k} \right\}_q$ were given in [6] a combinatorial interpretation in terms of weighted partitions. Therefore new q-Bell numbers introduced above seem to deserve attention. In [7] a family of the so called ψ-Poisson processes was introduced. The corresponding choice of the function sequence ψ leads to the q-Poisson process.

References

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